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A Formal Statement of the Einstein–Podolsky–Rosen Argument

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Assumptions of the original Einstein–Podolsky–Rosen argument are formally stated as axioms and nonlogical rules of inference. Then the argument is formally stated, making explicit the assumptions, logical structure, and conclusions involved. In turn several interpretative disputes are resolved. One frequent objection to the argument and a prevalent response to that objection as well are shown to be misguided.

1. INTRODUCTION

Two important facts characterize the well-known Einstein–Podolsky–Rosen (EPR) argument of 1935. First, it enjoys renewed prominence today, for recent foundational studies have motivated reevaluation of the original argument. Second, there is no accord among researchers concerning essential aspects of the argument. Because of this renewed interest and lack of consensus, in this paper I will formally reproduce the EPR argument in a way that graphically displays the logical structure of the argument.

The EPR argument is again in the news, for at least the following three reasons. Some researchers, notably C. A. Hooker (1970, 1972), have begun to use the EPR argument as a challenge to provide a physically meaningful and consistent interpretation of quantum theory. Others, including Josef Jauch (1968), Henry Krips (1969), and P. F. Zweifel (1974), have evaluated quantum measurement theories according to their ability to handle the physical situation described by EPR. Bas C. van Fraassen (1974) has pointed out that the EPR paradox might function as a test for the adequacy of quantum logics.

But there is no consensus about the important features of the EPR argument. Despite frequent criticism, detractors have been able to find *no common ground for dissent*, a fact noted by Einstein himself [reported in Max Jammer (1974, p. 187)]. Einstein apparently was piqued to find a number of physicists, each with a different objection, pointing out *the* mistake of the argument. Krips (1969, p. 149) has registered amusement that philosophers faced with the argument resort to mathematics and physicists to philosophy.

More serious is the *lack of consensus regarding assumptions* on which the argument rests. For conflicting views see Hooker (1972), Jammer (1966, 1974), E. Scheibe (1973), and B. H. Kellett (1977).

Disagreement concerning logical status is crucial. One extreme view characterizes the argument as fallacious: L. Rosenfeld (1968), Hooker (1972), J. L. B. Cooper (1950), Edwin Kemble (1935), and Kellett (1977). However, Hooker thinks that the argument, defective as stated, can be repaired by tinkering; the others, except Rosenfeld who offers no support for his view, apparently have failed to distinguish between validity (good logical structure) and soundness (validity plus premise truth). Kemble [see Jammer (1974, p. 193)] later clarified his charge, allowing that the argument was valid while possibly resting on false premise(s).

Among commentators who regard the argument valid there are additional controversies. For example, is there an *EPR contradiction*? L. E. Ballentine (1970, p. 362), Karl Popper (1968, p. 203; 1967, p. 28) and Donald Reisler (1971, p. 830) flatly deny the possibility. But Jeffrey Bub (1974), Kellett (1977), Scheibe (1973), Zweifel (1974), Krips (1969), and René Dugas (1936) each claimed to have located a contradiction proven by the EPR argument. (Regrettably these commentators have found different contradictions.) A moderate view characterizes the argument as paradoxical (leading to a strange or surprising result) without stressing the question of a paradox (leading to a contradiction): van Fraassen (1974), O. R. Frisch (1971), J. S. Bell (1964), as well as Cooper (1950), Henry Margenau (1936), and Schrodinger (1935).

One final dispute concerns the *import of the EPR argument*. The majority position regarding the argument as a proof that quantum theory is incomplete is nicely stated by Ballentine (1970) and Bub (1974). An alternate interpretation first held by W. H. Furry (1936) and recently defended by Jauch (1968) and Kellett (1977) regards the EPR argument as an empirical alternative to orthodox quantum theory. A still different point of view has been offered by Bub (1974) and Popper (1959), who argue that EPR were objecting to a particular interpretation of the Heisenberg uncertainty principle. Arthur Ruark (1935) and Cooper (1950) have stressed that EPR were pleading for a realistic interpretation of quantum theory.

Not all these views are mutually conflicting. In fact, examination of the formal statement which follows shows that most of these interpretations have at least some measure of truth.

2. FORMAL PRELIMINARIES

We shall develop the formal statement as follows. First key concepts and propositions are assigned abbreviations. Then axioms and rules are stated. Unlike a strictly logical proof, the EPR derivation relies on a physical basis, and it is the axioms and nonlogical rules that incorporate this basis. Finally, the argument itself is formally reproduced as a proof from the axioms in accord with the rules.

First, axioms are introduced to represent propositions that EPR intend to be acceptable to those who accept orthodox quantum theory. These axioms may appear to be premises of the argument: they are listed at the head of proofs and later reiterated into those proofs. However, for reasons that will become apparent, axioms are distinguished from premises by their physical status as axioms, by their placement in the proofs, and by their numbering with Roman numerals.

Second, nonlogical rules are employed. These rules, like their strictly logical counterparts, justify the movement from any nonpremise line of the derivation to a subsequent line. And like logical rules of inference, these rules assert that given such and such it is permissible to assert such and such. Unlike their logical counterpart, however, nonlogical rules are justified just as the physical axioms were: EPR thought them acceptable to those who accept orthodox quantum theory (possible exceptions are noted later).

The logical rules that are required are basic rules of the propositional calculus. See Fitch (1952).

Let us begin the formal treatment by stipulating the abbreviations for the concepts used in the nonlogical rules. EPR themselves explicitly stated these rules in all cases except two (as noted later, two rules required in the formal derivation are tacit in the EPR text), but they did so informally, in prose. Therefore the following abbreviations are required for this formal treatment; they are not used in the original EPR text.[1]

Concept Abbreviations Used in Rules

Observables: $\mathcal{O}$
Operators: $\hat{O}$
Eigenfunctions: θ, ϕ, ξ, Φ
Eigenvalues: o

[1] When I state the nonlogical rules, I use my own abbreviations as noted in the text. When I state the formal argument, I reproduce the actual EPR notation (the only exception is that I distinguish $\hat{A}$ and $\mathcal{A}$ whereas EPR use A in both cases).

Systems: S

Relations among concepts are also abbreviated.

Relation Abbreviations

Represents: {Rep}

Corresponds to: {Cor}

Is isolated from and without physical interaction with: {Iso}

Is not disturbed by: {ND}

Several propositional constants abbreviate entire propositions that are used repeatedly in the EPR argument.

Propositional Constants

C	The theory (or wave function) gives a complete description of physical reality (or the state).[2]
$CP^{R^{\mathcal{O}}}$	There is a counterpart (of an element of reality corresponding to observable $\mathcal{O}$) in the wave function (or theory).
P_o	Observable $\mathcal{O}$ is predicted with certainty to have value o.
$R^{\mathcal{O}}$	There is an element of reality corresponding to observable $\mathcal{O}$.
$M_S{}^{\mathcal{O}}$	A measurement is made on observable $\mathcal{O}$ in system S.
$*^{\hat{O}^1\hat{O}^2}$	Operators $\hat{O}^1$ and $\hat{O}^2$ are noncommuting.
$M_S{}^{\mathcal{O}} \rightarrow o_k$	As a result of measurement some o_k will be recorded.

Time indications are as follows: $t < 0$ is a time when systems are isolated, $0 \leqslant t \leqslant T$ is a time when systems interact, and $t > T$ is a time when once-interacting systems are isolated.

In addition to these abbreviations, I use the following standard logical symbols: $\cdot$ (conjunction), $\vee$ (disjunction), $\sim$ (negation), $\Diamond$ (the possibility modal operator), $\supset$ (implication), as well as parentheses and brackets. Mathematical symbols include: $=$ (equality), $\sum$ (summation), and $<$ (inequality). Having stipulated the terminology, we can begin the formal statement.

3. THE FORMAL STATEMENT

The EPR argument is a three-stage argument. Stages I and II support Stage III; they prove theorems required by Stage III. Stage I has the structure of a categorical proof concluding with Theorem I.10 which is read 'It is not the case that there is an element of reality corresponding to observable $\mathcal{P}$ and an element of reality corresponding to observable $\mathcal{Q}$ when operators $\hat{P}$ and $\hat{Q}$ are noncommuting, or it is not the case that quantum theory gives a complete description of physical reality'.

[2] Regrettably EPR equate two notions of completeness: "complete representation by a wave function" and "complete theory" are used interchangeably. Yet it appears that no harm, beyond lack of clarity, results. I simply use C to refer to both senses of completeness.

Stage II is also a categorical proof, one that proves Theorem II.21, read 'If the theory gives a complete description of physical reality, then there is an element of reality corresponding to observable $\mathscr{P}$ of System II and an element of reality corresponding to observable $\mathscr{Q}$ of System II and operators $\hat{P}$ and $\hat{Q}$ are noncommuting'.

These two theorems play an important role in Stage III, a *reductio ad absurdum* concluding $\sim C$, 'Quantum theory does not give a complete description of physical reality'. Let us now consider each stage in more detail.

Stage I is a ten-step argument that relies on one physical axiom and four nonlogical rules. The axiom formalizes the EPR description of the behavior of a particle having a single degree of freedom. In the system EPR describe, wave function ψ characterizes the state of the particle. And corresponding to momentum and position observables $\mathscr{P}$ and $\mathscr{Q}$ are operators $\hat{P}$ and $\hat{Q}$. ψ is an eigenfunction of $\hat{P}$, so one obtains the axiom

$$\hat{P}\{\mathrm{Cor}\}\mathscr{P}\cdot\hat{Q}\{\mathrm{Cor}\}\mathscr{Q}\cdot\hat{P}\psi = p\psi \tag{I.i}$$

which is stated by EPR at 778a [all references are to EPR (1935); a and b indicate left and right columns].

The first of the four nonlogical rules is the EPR requirement of completeness for a physical theory. The rule is stated by EPR at 777b, reiterated in the abstract, and applied at 778b.

Nonlogical Rule: RC

$$\begin{array}{l|l} & C \\ \hline & \begin{array}{l|l} & R^{\mathcal{O}} \\ \hline & CP^{R^{\mathcal{O}}} \end{array} \end{array}$$

The horizontal lines mark off hypotheses of the statement. Accordingly, this rule may be read 'If quantum theory is a complete description of physical reality, then if there is an element of reality corresponding to observable $\mathcal{O}$, then there is a counterpart of that element of reality in the theory'.

The second rule used in Stage I reflects the EPR claim that if a theory contains a counterpart of an element of reality, then the possibility of the predictability of the value of the observable corresponding to that element of reality is insured (see 778b).

Nonlogical Rule: Z

$$\begin{array}{l|l} & CP^{R^{\mathcal{O}}} \\ \hline & \Diamond P_{o} \end{array}$$

The final two nonlogical rules capture features of orthodox quantum theory. One is tacit in the EPR text [but see the relevant discussion of equations (1)–(6) at 778].

Nonlogical Rule: X

$$\begin{array}{|l} *^{\hat{O}_1\hat{O}_2} \\ \hat{O}^1\theta^1 = o_1\theta^1 \\ \hline \sim(\hat{O}^2\theta = o\theta) \end{array}$$

If one quantifies in a physically obvious manner, Rule X may be paraphrased as follows: 'If operators $\hat{O}^1$ and $\hat{O}^2$ do not commute and if for some eigenstate θ^1 operator $\hat{O}^1$ has eigenvalue o_1, then for any eigenstate θ it is not the case that $\hat{O}^2$ has an eigenvalue o in state θ'. The other rule denies the possibility of predicting values of observables whose operator is not in an eigenvalue equation.

Nonlogical Rule: Y

$$\begin{array}{|l} \sim(\hat{O}\theta = o\theta) \\ \hline \sim\Diamond P_o \end{array}$$

This rule is stated at 778a: "On the other hand if $A\psi = a\psi$ does not hold, we can no longer speak of the physical quantity $\mathscr{A}$ having a particular value... we can only say that the relative probability...." Since 'P_o' represents 'Observable $\mathscr{O}$ is predicted with certainty to have value o', the formalization is faithful to the text.

Stage I begins with the axiom I.i and, using the four nonlogical rules just cited, goes on to prove line I.10, the conclusion of the argument.[3] EPR summarize Stage I at 778b.

Stage I

i	$\hat{P}\{\text{Cor}\}\mathscr{P}\cdot\hat{Q}\{\text{Cor}\}\mathscr{Q}\cdot\hat{P}\psi = p\psi$	
1	$(R^{\mathscr{P}}\cdot R^{\mathscr{Q}}\cdot *^{\hat{P}\hat{Q}})\cdot C$	*Hypothesis*
2	$CP^{R^{\mathscr{P}}}$	1, RC
3	$\Diamond Pp$	2, Z
4	$CP^{R^{\mathscr{Q}}}$	1, RC
5	$\Diamond P_q$	4, Z
6	$\Diamond Pp\cdot\Diamond Pq$	3, 5, conjunction introduction
7	$\sim(\hat{Q}\psi = q\psi)$	*i*, 1, X
8	$\sim\Diamond Pq$	7, Y
9	$\sim[(R^{\mathscr{P}}\cdot R^{\mathscr{Q}}\cdot *^{\hat{P}\hat{Q}})\cdot C]$	5, 8, *reductio ad absurdum*
10	$\sim(R^{p}\cdot R^{q}\cdot *^{\hat{P}\hat{Q}})\vee\sim C$	9, de Morgan's Law

[3] The *reductio ad absurdum* of Stage I is proved by establishing steps 6 and 8 as contradictory. A shorter *reductio* would conjoin steps 5 and 8, thereby eliminating steps 2, 3, and 6. The *reductio* here presented, while longer, is more graphic and textually faithful.

Stage I is a categorical argument, a *reductio ad absurdum* of the conjunction posited as a hypothesis in line 1 and denied in line 10. Notice that line i is an axiom, not a hypothesis of the argument. Line 10 is a theorem that will be used in Stage III.

Stage II is a much longer argument, one that subtly combines physics, interpretative rules, and logic. In fact, much of the dispute among commentators of EPR has arisen from a failure to grasp the logical structure of the Stage II argument. Accordingly this section stresses its logical structure and gives little emphasis to the physical examples that motivate the argument.

Stage II

i	$u\{\text{Rep}\}S^{\mathrm{I}}t = 0\cdot v\{\text{Rep}\}S^{\mathrm{I}}t = 0\cdot \hat{A}\{\text{Cor}\}\mathscr{A}^{\mathrm{I}}\cdot \hat{B}\{\text{Cor}\}\mathscr{B}^{\mathrm{I}}\cdot \hat{B}v_s = b_s v_s\cdot \hat{A}u_n = a_n u_n$	
ii	$S^{\mathrm{I}}\{\text{Iso}\}S^{\mathrm{II}}$	
iii	$\psi\{\text{Rep}\}S^{\mathrm{II}}t = 0\cdot\varphi\{\text{Rep}\}S^{\mathrm{II}}t = 0\cdot\hat{P}\{\text{Cor}\}\mathscr{P}^{\mathrm{I}}\hat{Q}\{\text{Cor}\}\mathscr{Q}^{\mathrm{I}}\cdot *^{\hat{P}\hat{Q}}$	
iv	$\hat{U}\Psi_{t=0} = \Psi_{t=t>T}$	
v	$\Diamond M^{\mathscr{A}}_{\mathrm{SI}}\cdot\Diamond M^{\mathscr{B}}_{\mathrm{SI}}$	
1	C	*Hypothesis*
2	$\Psi\{\text{Rep}\}S^{\mathrm{I+II}} \qquad t > T$	i, iii, iv, CM
3	$\Psi = \sum_n \psi_n u_n\{\text{Rep}\}S^{\mathrm{I+II}}$	2, B
4	$\Diamond M^{\mathscr{A}}_{\mathrm{SI}}$	v, conjunction elimination
5	$M^{\mathscr{A}}_{\mathrm{SI}} \rightarrow a_n$	i, 4, M
6	$\Diamond(\psi_k\{\text{Rep}\}S^{\mathrm{II}})$	i, 2, 3, 4, 5, RWP
7	$\Psi = \sum_s \varphi_s v_s\{\text{Rep}\}S^{\mathrm{I+II}}$	2, B
8	$\Diamond M^{\mathscr{B}}_{\mathrm{sI}}$	v, conjunction elimination
9	$M^{\mathscr{B}}_{\mathrm{sI}} \rightarrow b_s$	i, 8, M
10	$\Diamond(\varphi_r\{\text{Rep}\}S^{\mathrm{II}})$	i, 2, 7, 8, 9, RWP
11	$\Diamond(\psi_k\{\text{Rep}\}S^{\mathrm{II}})\cdot\Diamond(\varphi_r\{\text{Rep}\}S^{\mathrm{II}})$	6, 10, conjunction introduction
12	$M_{\mathrm{SI}}\{\text{ND}\}S^{\mathrm{II}}$	ii, I
13	$\Diamond P_{p\mathrm{SII}}\cdot P_{p\mathrm{SII}}\{\text{ND}\}S^{\mathrm{II}}$	3, 12, W
14	$R^{\mathscr{P}}_{\mathrm{sII}}\{\text{Cor}\}\mathscr{P}$	13, CR
15	$\Diamond P_{q\mathrm{SII}}\cdot P_{q\mathrm{SII}}\{\text{ND}\}S^{\mathrm{II}}$	7, 12, W
16	$R^{q}_{\mathrm{SII}}\{\text{Cor}\}\mathscr{Q}$	15, CR
17	$CP^{R\mathscr{P}}_{\mathrm{SII}}$	1, 14, RC
18	$CP^{R\mathscr{Q}}_{\mathrm{sII}}$	1, 16, RC
19	$CP^{R\mathscr{P}}_{\mathrm{sII}}\cdot CP^{R\mathscr{Q}}_{\mathrm{sII}}$	17, 18, conjunction introduction
20	$R^{\mathscr{P}}_{\mathrm{sII}}\cdot R^{\mathscr{Q}}_{\mathrm{sII}}\cdot *^{\hat{P}\hat{Q}}$	iii, 14, 16, conjunction introduction
21	$C \supset (R^{\mathscr{P}}_{\mathrm{SII}}\cdot R^{\mathscr{Q}}_{\mathrm{SII}}\cdot *^{\hat{P}\hat{Q}})$	1–20, implication introduction

Five axioms are required for Stage II. The first and third, (II.i) and (II.iii), describe specific physical systems, as did axiom I.i for Stage I. These axioms are straightforward formal abbreviations of statements about the physics of interacting quantum mechanical systems. Axiom ii simply asserts that systems S^{I} and S^{II} are physically isolated (at $t > T$), as EPR demand at 779a. Axiom iv specifies a time evolution operator (779a). Axiom v asserts that it is possible to measure observable $\mathscr{A}$ of system S^{I} and it is possible to measure observable $\mathscr{B}$ of system S^{I}. Comments on this axiom are included in the following section.

Obviously nonlogical rules play an important role in Stage II. The first rule used is CM, a recipe for writing the representation of the state of a combined system having been given the representation of states of two component systems and the relevant time evolution operator. This rule reflects comments from the first full paragraph of 779a. Notice that neither the EPR text nor the formal rule asserts that the combined system is in a pure state.

Nonlogical Rule: CM

$$\begin{array}{|l} \xi\{\mathrm{Rep}\}S^{1} \quad (t = 0) \\ \theta\{\mathrm{Rep}\}S^{2} \quad (t = 0) \\ \underline{\hat{U}\Phi_{t=0} = \Phi_{t=t>T}} \\ \Phi\{\mathrm{Rep}\}S^{1+2} \quad (t > T) \end{array}$$

The second rule formalizes EPR equations (7) and (8) of 779, equations that "expand" a wave function into a series of orthogonal functions.

Nonlogical Rule: B

$$\begin{array}{|l} \underline{\Phi\{\mathrm{Rep}\}S^{1+2}} \\ \Phi = \sum_{n} \xi_n \theta_n\{\mathrm{Rep}\}S^{1+2} \end{array}$$

The next rule required by the argument is tacitly assumed by EPR. The rule straightforwardly asserts that if an observable $\mathcal{O}$ has an operator $\hat{O}$ and an eigenvalue equation $\hat{O}\theta = o\theta$, and if it is possible to measure the observable $\mathcal{O}$ in some system S, then as a result of the measurement some value of $\mathcal{O}$, o_k, will be recorded.

Nonlogical Rule: M

$$\begin{array}{|l} \hat{O}\theta = o\theta \\ \underline{\Diamond M_S^{\mathcal{O}}} \\ M_S^{\mathcal{O}} \rightarrow o_k \end{array}$$

Rule RWP is a formal version of "the process of reduction of the wave packet" as described by EPR at 779a.

Nonlogical Rule: RWP

$$
\begin{array}{|l}
\hat{O}\theta = o\theta \\
\Phi\{\text{Rep}\}S^{1+2} \\
\Phi = \sum_n \xi_n\theta_n\{\text{Rep}\}S^{1+2} \\
\Diamond M_{S^1}^{O} \\
M_S^{O} \rightarrow o_k \\
\hline
\Diamond(\theta_k\{\text{Rep}\}S^1) \\
\Diamond(\xi_k\{\text{Rep}\}S^2)
\end{array}
$$

Later we shall comment briefly about Rule RWP; here we note only that EPR explicitly used the rule in their argument.

The second division of Stage II begins at II.12 with the application of Rule I, which reflects EPR's definition (779b) of "absence of an interaction between the two systems." This rule is the basis of many discussions about EPR and locality.

Nonlogical Rule: I

$$
\begin{array}{|l}
S^1\{\text{Iso}\}S^2 \\
\hline
M_{S^1}\{\text{ND}\}S^2
\end{array}
$$

The next rule captures an insight of EPR that makes the Stage II argument significant. If two systems are such that a measurement performed on one does not disturb the second and if the state of the combined system is represented by a wave function, then it is possible to predict the value of an observable of the second system and, of course this prediction can be made without disturbing the second system. This situation, discussed by EPR on 779, is formalized by Rule W.

Nonlogical Rule: W

$$
\begin{array}{|l}
\Phi = \sum_n \xi_n\theta_n\{\text{Rep}\}S^{1+2} \\
M_{S^1}^{O}\{\text{ND}\}S^2 \\
\hline
\Diamond P_{oS^2}\cdot P_{oS^2}\{\text{ND}\}S^2
\end{array}
$$

The final rule that EPR use is the one that has attracted the most notice. EPR themselves emphasize the rule by beginning the article (777b, see also the abstract) with its statement. The rule summarizes the EPR criterion of physical reality.

Nonlogical Rule: CR

$$
\begin{array}{|l}
\Diamond P_o \\
P_o\{\mathrm{ND}\}S \\
\hline
R^{\mathcal{O}}\{\mathrm{Cor}\}\mathcal{O}
\end{array}
$$

The rule is applied at 778a, 779b, and 780b.

Stage III is relatively straightforward, especially from the physical point of view. No axioms or nonlogical rules are required. Logically the argument requires just six steps. Here for clarity we have dropped superscripts that were necessary in earlier stages.

Stage III

1	C	*Hypothesis*
2	$C \supset (R \cdot R \cdot *)$	Theorem, II.21
3	$(R \cdot R \cdot *)$	1, 2, *modus ponens*
4	$\sim C \vee \sim(R \cdot R \cdot *)$	Theorem, I.10
5	$\sim C$	3, 4, *modus tollendo ponens*
6	$\sim C$	1–5 *reductio ad absurdum*

Notice that lines 2 and 4 are reiterations of the theorems that were the conclusions of Stages I and II. Textual support for Stage III is unequivocally provided by the first complete paragraph of 780b where the argument is stated in the article's summary. See also the abstract.

4. COMMENTS ON THE FORMAL STATEMENT

At the end of Section 1 I suggested that the formal statement might clear up some of the interpretative disputes that characterize EPR commentary. Let us now briefly look at these issues in the light of the formal statement.

The *lack of consensus concerning assumptions* can be explained as follows. The incompleteness proof is categorical, i.e., it requires no hypotheses in its main proof and uses only theorems that are categorically proved. Others have claimed that the argument is hypothetical, requiring Rule CR as a hypothesis. So construed, the proof would conclude

If Rule CR then not C

An equivalent formulation is [see Ballentine (1970, p. 363)]

Not both Rule CR and C

While EPR would undoubtedly not have favored such an interpretation [see the EPR text, 777, or Podolsky's remark in Jammer (1974, p. 193)], it is logically admissible to so state the argument. In fact there is no logical prohibition against regarding *all* the axioms and nonlogical rules as hypotheses of the argument; and it is more or less arbitrary to single out one or another of them. Of course any decision to highlight particular rules or axioms imparts a nuance to the argument.

The *dispute concerning the logical status* of the argument is resolved in agreement with the majority view that the argument is valid. For if the axioms and nonlogical rules are admitted, the conclusion can be deductively obtained by the canons of classical propositional logic.

The question of the *EPR contradiction* necessitates an additional look at the formal statement. EPR promise (779) to show that once completeness is assumed (along with the nonlogical rules and axioms) a contradiction can be derived. They make good on their promise. Consider

$$\Diamond(\psi_k\{\mathrm{Rep}\}S^{\mathrm{II}})\cdot\Diamond(\varphi_r\{\mathrm{Rep}\}S^{\mathrm{II}}) \tag{II.11}$$

$$CP_{S\mathrm{II}}^{R\mathscr{P}}\cdot CP_{S\mathrm{II}}^{R\mathscr{Q}} \tag{II.19}$$

$$R_{S\mathrm{II}}^{\mathscr{P}}\cdot R_{S\mathrm{II}}^{\mathscr{Q}}\cdot *^{\hat{P}\hat{Q}} \tag{II.20}$$

These three lines, proved in Stage II, are contradictory, though not obviously so. In the text corresponding to (II.11), EPR claim that two wave functions are assigned "to the same reality (the second system)." Accordingly, substitute the constant '$\mathbb{R}$' for S^{II} in (II.11). And since wave functions are the counterparts in the theory, substitute CP for the wave functions ψ_k and φ_r to obtain

$$CP^{\psi}\{\mathrm{Rep}\}\mathbb{R}\cdot CP^{\varphi}\{\mathrm{Rep}\}\mathbb{R} \tag{II.11$'$}$$

At this point the contradiction is implicit, for two wave functions represent the same reality (many who find the EPR argument paradoxical have this fact in mind). To make the contradiction explicit, compare (II.11′) to (II.19) and (II.20): for the same physical situation wave functions represent both one and two (not-one) element(s) of reality. Dugas (1936), Krips (1969), and Zweifel (1974) cite a similar but different contradiction.

Concerning the *import of the EPR argument*, note first how the EPR incompleteness proof (the main three-stage argument) is related to the EPR contradiction just sketched. The entire contradiction argument takes place in Stage II under the hypothesis of completeness (II.1). But EPR eventually reject that hypothesis; that is just the force of their entire proof. So for EPR themselves there is no contradiction; the contradiction arises only for those who insist on completeness. To put the same point another way, the EPR contradiction is a *reductio ad absurdum* of the completeness assumption. Since the entire three-stage EPR argument is a proof of incompleteness, the

EPR incompleteness proof and the EPR contradiction are opposite sides of the same coin.

A final comment concerns *EPR detractors*, all of whom may be classified according to their attempts to attack particular EPR rules and axioms. For example Wolfe (1935), Bohr (1935), Epstein (1945, p. 136), and Costa de Beauregard (1965, 1976) would all likely deny Rule I, each for a different reason. Frisch (1971), Putnam (1961), and Sharp (1961) have denied the truth of the antecedent of Rule I, i.e., axiom II.ii. And of course Margenau (1936) and others deny Rule RWP. But Reisler (1967) shows that the paradox can be restated avoiding Rule RWP. For a discussion of Rule CM, see Furry (1936), Jauch (1968), Reisler (1971), Ballentine (1970, p. 370 n. 13), and Gardner (1972, p. 107 n. 1). Any objection to EPR must be an objection to some axiom or nonlogical rule; and consequently the formal statement is the grounds for a taxonomy of objections.

5. SIMULTANEOUS MEASUREMENT OBJECTIONS AND THE ALREADY-EXISTED STRATEGY

One distinguishing feature of orthodox quantum mechanics is the restriction placed on simultaneous measurements [see Mandl (1960, sec. 19)]: if operators corresponding to two observables do not commute, then the two observables are not simultaneously measurable. The relevance to EPR is as follows. Axiom iii of Stage II asserts that $\hat{P}$ and $\hat{Q}$ are noncommuting operators. Therefore, by orthodox quantum theory, observables $\mathscr{P}$ and $\mathscr{Q}$ cannot be simultaneously measured and if the EPR argument assumes that they can be, then the argument requires a false assumption. Just this objection—that the EPR argument is unsound because it presupposes simultaneous measurement—has been regarded by many researchers as fatal to the EPR position. For example, Ruark (1935), Wolfe (1935), Bohr (possibly in 1935, more clearly in 1948), Dickie and Wittke (1960), Bohm (1961), and Schlegel (1970) have all thought that the objection undercut the EPR argument.

The formal statement makes clear the EPR position and shows that nowhere is it required to assert that incompatible measurements are simultaneously made. The crucial step is line II.20, a step that asserts simultaneous existence of two elements of reality. Those who have objected have thought that line II.20 is derived from an assertion that the measurements (of II.4 and II.8) must be simultaneously carried out. However, EPR assert only that each measurement is possible. "Suppose now that the quantity $\mathscr{A}$ is measured... [779a]" and "If, instead of this, we had chosen another quantity, say $\mathscr{B}$... [779b]". EPR need not have intended that both $\mathscr{A}$ and $\mathscr{B}$ be measured. Specifically they do not require an axiom such as the following.

$$M_{SI}^{\mathscr{A}} \cdot M_{SI}^{\mathscr{B}} \qquad \text{(II.v')}$$

Rather they require the weaker modal version actually used in the formal statement

$$\Diamond M_{SI}^{\mathscr{A}} \cdot \Diamond M_{SI}^{\mathscr{B}} \tag{II.v}$$

When the EPR text is read this way, the simultaneous measurement objection simply fails.

A second confusion has arisen around the issue of simultaneous measurement. Both friends of EPR such as van Fraassen (1974, p. 239) and Bub (1974, p. 41) and detractors such as Kellett (1977) have thought that the EPR position is made reasonable only by a strategy that EPR describe in their penultimate paragraph (780b). There EPR consider and reject the possible objection (Copenhagen in nature) that the EPR criterion of reality is "not sufficiently restrictive" because it allows reality independent of measurement, while a more restrictive criterion would allow reality to observables only in case the observables could be measured or predicted. EPR base their rejection of the more restrictive criterion on the alleged dependence of reality on measurement. Specifically, suppose that one were to adopt the more restrictive criterion for the EPR experiment. Then if the experimenter chose one measurement, it would seem that he would have one reality, while if he chose another measurement he would have another reality. Of course EPR found this unacceptable, as did Schrodinger (1935, p. 559), for they shared a 'must have existed before measurement' conviction.

If this conviction is made into an assumption, as Bub, van Fraassen, Kellett, and others apparently seem to think is necessary, then EPR might avoid the simultaneous measurement objection; for either of two measurements could be made at the will or choice of the experimenter. And if reality is independent of measurement, then the elements of reality that would be discovered by the chosen measurement 'must have already existed' before measurement.

Although it may be reasonable to make such an assumption and even to attribute it to EPR, it would be a mistake to think the 'must have existed before measurement' conviction is a part of the actual EPR argument. As we have shown, EPR nowhere require simultaneous measurement. Consequently they need not be defended against simultaneous measurement objections with the 'must have existed before measurement' conviction. Whatever the status of that conviction, it is not required by the actual EPR argument, as the formal statement graphically shows. A fuller account of the issues discussed in this section is given in McGrath (1977).

6. CONCLUDING REMARKS

We reserve this conclusion for comments concerning the uses and limitations of the formal statement. Because some concepts, relations, and propositions are taken as unanalyzed primitives, the task of "unpacking" such notions as "physical reality" and "counterpart in a physical theory" remains [as an example of this type of investigation, see the treatment by Moldauer (1974)]. The type of analysis carried on here is, in itself, incapable of resolving such conceptual problems. Note, however, that solutions to the problem of "unpacking" concepts presuppose grasping the formal structure of the argument. A second limitation of the formal analysis concerns inability to evaluate physical reasonableness of axioms and rules and the assessment of possible experimental consequence of the EPR position. For a recent example of such an investigation see Fortunato (1977). Again, while no formal analysis can be the final arbiter of these questions, the physical assessment cannot be satisfactorily carried out until the formal structure of the argument has been apprehended.

If the formal statement accurately reproduces the EPR argument—and care has been taken to cite EPR text corresponding to crucial parts of the formal statement to show that it does—then here laid bare for all to see, defenders and detractors as well, is the structure of the Einstein–Podolsky–Rosen argument. Despite the critical disputes, the EPR text is unequivocal, as unequivocal as English prose interlaced with mathematical and physical formulas can be. Once the structure of the argument is seen in the text, the formal statement loses importance. Until it is seen, critical disputes cannot be resolved and the formal statement is indispensable.

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